If $a^x = N$, then $x = \log_a N$ (x equal to log N base a). There are two log bases commonly used: base 10 and base e which is also known as natural logarithm. Natural or e based log is also written as ln.

Properties of logarithm:

$\log_a(MN) = \log_a M + \log_a N$

$\log_a \frac{M}{N} = \log_a M - \log_a N$

$\log_a a = 1$

$\log_a a^x = x \log_a a = x$

$\log_a 1 = \log_a a^0 = 0$ [For any base]

$\log_a b = \frac{\log_x b}{\log_x a}$

Problem: Solve the equation $\log_{10} x^2 + \log_{10} x = 3$

$\log_{10} x^2 + \log_{10} x = 3$

=> $\log_{10}(x^2 x) = 3$

=> $x^3 = 10^3$

=> $x = 10$

Problem: Calculate approximate value of the following expression without using calculator

$\log_{10} 370 + \log_{10} 1020 + \log_{10} 130$

= $\log_{10}(3.70 \times 10^2) + \log_{10}(1.02 \times 10^3) + \log_{10}(1.30 \times 10^2)$

= $\log_{10} 3.70 + \log_{10} 10^2 + \log_{10} 1.02 + \log_{10} 10^3 + \log_{10} 1.30 + \log_{10} 10^2$

= $\log_{10} 3.70 + 2 + \log_{10} 1.02 + 3 + \log_{10} 1.30 + 2$

≅ 7 [log value less than 10 with base 10 is less than 1, so fractional parts are omitted]

Problem: Solve the equation $3 \log_2 x + 2 \log_2 x - 2 \log_2 x = 3$

$3 \log_2 x + 2 \log_2 x - 2 \log_2 x = 3$

=> $\log_2 x^3 + \log_2 x^2 - \log_2 x^2 = 3$

=> $\log_2(x^3 x^2) - \log_2 x^2 = 3$

=> $\log_2 \frac{x^5}{x^2} = 3$

$\Rightarrow \log_2 x^3 = 3$

$\Rightarrow x^3 = 2^3 = 8$

$\Rightarrow x = 8^{\frac{1}{3}} = 2$